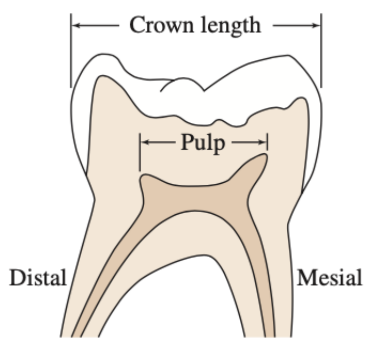
Problem to be used on 9/22:
3.1 (Limits), 3.2 (Continuity), 3.3 (Rates of Change)

Group 1

Application to Life Sciences The crown length (as shown below) of first molars in $L(t) = -0.01t^2 + 0.788t - 7.048$, where $L(t)$ is the crown length, in millimeters, of the molar t weeks after conception.



(a) Find the average rate of growth in crown length during weeks 22 through 28.
(b) Find the instantaneous rate of growth in crown length when the tooth is exactly 22 weeks of age.

Group 2

Application to Business and Economics Suppose that the total profit in hundreds of dollars from selling x items is given by

$$P(x) = 2x^2 - 5x + 6$$

(a) Find the average rate of change of profit for x from 2 to 3.
(b) Find the instantaneous rate of change of profit with respect to the number of items produced when x = 2. (This number is called the marginal profit at x = 2.)

Group 3

Find the value of the constant k that makes the function continuous.

$$f(x) = \begin{cases} \frac{3x^2+2x-8}{x+2} & x \neq -2 \\ 3x + k & x = -2 \end{cases}$$

Group 4

Decide whether each limit exists. If a limit exists, find its value.
(a)

$$\lim_{x\to 3} \frac{x^2 - 9}{x - 3}$$

(b)

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

(c)

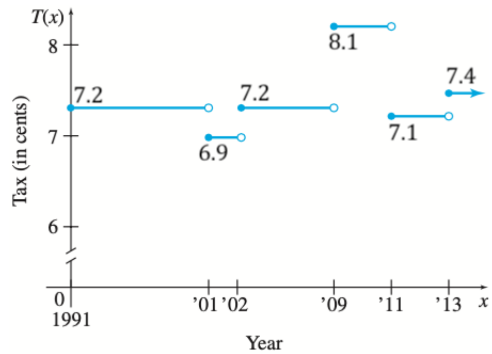
$$\lim_{x\to \infty} \frac{x^2 + 3x + 2}{2x^2 - x}$$

(d)

$$\lim_{x\to \infty} \frac{x^3 - 9x^2 + x - 2}{x^2 - 3x + 1}$$

Group 5

Officials in California tend to raise the sales tax in years in which the state faces a budget deficit and then cut the tax when the state has a surplus. The graph below shows the California state sales tax in recent years. Let $T(x)$ represent the sales tax per dollar spent in year x. Find the following.



(a) $\lim_{x\to 94} T(x)$
(b) $\lim_{x\to 13^-} T(x)$
(c) $\lim_{x\to 13^+} T(x)$
(d) $\lim_{x\to 13} T(x)$
(e) $T(13)$

Extra Problems

1. **Application to Environmental Sciences**
   To develop strategies to manage water quality in polluted lakes, biologists must determine the depths of sediments and the rate of sedimentation. It has been determined that the depth of sediment $D(t)$ (in centimeters) with respect to time (in years starting from 1990, t=0 means year 1990) for Lake Coeur dAlene, Idaho, can be estimated by the equation
   $$D(t) = 155(1 - e^{-0.0133t})$$
   (a) Find $D(20)$ and interpret.
   (b) Find $\lim_{t\to\infty} D(t)$ and interpret.

2. **Application to Business**
   Sales of snowblowers are seasonal. Suppose the sales of snowblowers in one region of the country are approximated by
   $$S(t) = 500 + 500\cos(\frac{\pi}{6}t)$$
   where t is time in months, with $t = 0$ corresponding to November. Find the sales for (a)(b)(c)(d)(e), and graph the function.
   (a) November, (b) January, (c) February, (d) May, (e) August

3. **Applications to Life Sciences**
   Researchers at Iowa State University and the University of Arkansas have developed

a piecewise function that can be used to estimate the body weight (in grams) of a male broiler during the first 56 days of life according to

$$W(t) = \begin{cases} 48 + 3.64t + 0.6363t^2 + 0.00963t^3 & 1 \leq t \leq 28 \\ -1004 + 65.8t & 28 < t \leq 56 \end{cases}$$

where t is the age of the chicken (in days).
(a) Determine the weight of a male broiler that is 25 days old.
(b) Is $W(t)$ a continuous function?
(Optional: (c) Comment on why researchers would use two different types of functions to estimate the weight of a chicken at various ages.)